Time reversal odd fragmentation functions in semi-inclusive scattering of polarized leptons from unpolarized hadrons

J. Levelt

Institut für Theoretische Physik, University of Erlangen-Nürnberg,
Staudtstrasse 7, D-91508 Erlangen, Germany

and

P.J. Mulders*

National Institute for Nuclear Physics and High Energy Physics
(NIKHEF-K), P.O. Box 41882, NL-1009 DB Amsterdam, the Netherlands

Abstract

Semi-inclusive deep inelastic scattering of polarized leptons off hadrons enables one to measure the antisymmetric part of the hadron tensor. For unpolarized hadrons this piece is odd under time reversal. In deep inelastic scattering it shows up as a $\langle \sin\phi \rangle$ asymmetry for the produced hadrons. This asymmetry can be expressed as the product of a twist-three "hadron → quark" distribution function and a time reversal odd twist-two "quark → hadron" fragmentation function. This fragmentation function can only be measured for nonzero transverse momenta of the produced hadron.

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* also at Department of Physics and Astronomy, Free University, Amsterdam

In this paper we study deep inelastic leptoproduction of hadrons. To be precise we investigate the process

$$\ell + H \longrightarrow \ell' + h + X, \tag{1}$$

where ℓ and ℓ' are the incoming and scattered lepton with momenta k and k', H is the hadronic target with mass M and momentum P, h is the produced hadron with mass M_h and momentum P_h and X is the unobserved rest of the final state. The differential cross section for this process is written as a product of a leptonic and hadronic tensor,

$$\frac{d\sigma}{dx_B\, dz\, dy\, d^2\boldsymbol{q}_T} = \frac{\pi\,\alpha^2\,y\,z}{2\,Q^4} L_{\mu\nu}\, 2M\mathcal{W}^{\mu\nu}, \tag{2}$$

where we have used the invariants expressed in the external momenta and the momentum q of the exchanged virtual photon (we will limit ourselves to electromagnetic interactions), which is spacelike and large ($-q^2 = Q^2 \gg M^2$). The invariants used are[1]

$$x_B = \frac{Q^2}{2\,P\cdot q}, \quad z = \frac{P\cdot P_h}{P\cdot q} \approx -\frac{2\,P_h\cdot q}{Q^2}, \quad y = \frac{P\cdot q}{P\cdot k}. \tag{3}$$

The (two-component) vector $\boldsymbol{q}_T$ is the transverse momentum of the photon in the frame in which the hadrons H and h are collinear. This translates to a perpendicular momentum of the produced hadron equal to $\boldsymbol{P}_{h\perp} = -z\boldsymbol{q}_T$ in the frame in which the target hadron H and the photon are collinear. The leptonic tensor for polarized leptons with helicity $\lambda = \pm 1$ is of the form

$$L_{\mu\nu}(k,k',\lambda) = \Big(2k_\mu k'_\nu + 2k_\nu k'_\mu - Q^2 g_{\mu\nu} + 2i\lambda\epsilon_{\mu\nu\rho\sigma}q^\rho k^\sigma\Big). \tag{4}$$

and the (unpolarized) hadronic tensor is of the form

$$\begin{aligned} 2M\mathcal{W}_{\mu\nu}(q,P,P_h) \;&=\; \frac{1}{(2\pi)^4}\int \frac{d^3P_X}{(2\pi)^3 2P_X^0}\,(2\pi)^4\delta^4(q+P-P_X-P_h) \\ &\qquad \times\langle P|J_\mu(0)|P_X,P_h\rangle\langle P_X,P_h|J_\nu(0)|P\rangle, \end{aligned} \tag{5}$$

where averaging over initial state spins and summation over final state spins is assumed.

In all of our considerations we will neglect all contributions of order $1/Q^2$, but keep all contributions of order $1/Q$, as it turns out that the time reversal odd fragmentation function, that we investigate, contributes at the $\mathcal{O}(1/Q)$ level. We will also not discuss the factorization of the process and the inclusion of radiative corrections. We assume that we can use the diagrammatic expansion for the hard process as outlined by Ellis, Furmánski and Petronzio (EFP) [1]. In this approach diagrams are separated in a hard scattering part and soft correlation functions [2].

The diagrammatic expansion of EFP is useful for deep inelastic scattering, because it in an appropriately chosen (color) gauge corresponds to an expansion in powers of Q^{-1}. This twist expansion as used here is discussed in refs [3, 4, 5, 6]. The Born diagrams in this expansion are given in Fig. 1 and give [5]

$$2M\,\mathcal{W}_{\mu\nu}(q,P,P_h) = \frac{1}{2}\int d^4p\, d^4k\, \delta^4(p+q-k)\, \mathrm{Tr}\,(\gamma_\mu\Phi(p)\gamma_\nu\Delta(k)) + \left\{ \begin{array}{c} q\leftrightarrow -q \\ \mu\leftrightarrow\nu \end{array}\right\}, \tag{6}$$

where the (soft) quark correlation functions [7, 3, 8] are given by

$$\Phi_{ij}(p) = \frac{1}{(2\pi)^4}\int d^4x\, e^{ip\cdot x}\,\langle P|\overline{\psi}_j(0)\psi_i(x)|P\rangle, \tag{7}$$

$$\Delta_{ij}(k) = \sum_X \frac{1}{(2\pi)^4}\int d^4x\, e^{ik\cdot x}\,\langle 0|\psi_i(x)|P_h,X\rangle\langle P_h,X|\overline{\psi}_j(0)|0\rangle. \tag{8}$$

[1] With an approximate equal sign we indicate relations valid up to order $1/Q^2$.

In order to analyze the result it is useful to use lightcone coordinates for the external momenta P, P_h and q and the integration variables p and k. Using the representation $p = [p^-, p^+, \boldsymbol{p}_T]$ where $p^\pm = (p^0 \pm p^3)/\sqrt{2}$ we write in a frame where H and h are collinear

$$\begin{aligned}
P &= \left[\frac{x_B M^2}{A\sqrt{2}}, \frac{A}{x_B\sqrt{2}}, \boldsymbol{0}_T\right], \\
p &= \left[p^-, \frac{A}{\sqrt{2}}, \boldsymbol{p}_T\right], \\
q &= \left[\frac{Q^2}{A\sqrt{2}}, -\frac{A}{\sqrt{2}}, \boldsymbol{q}_T\right], \\
k &= \left[\frac{Q^2}{A\sqrt{2}}, k^+, \boldsymbol{k}_T\right], \\
P_h &= \left[\frac{zQ^2}{A\sqrt{2}}, -\frac{AM_h^2}{zQ^2\sqrt{2}}, \boldsymbol{0}_T\right],
\end{aligned} \tag{9}$$

This parametrization is consistent with $p + q = k$ and has been made under the assumption that the quark momenta are limited to a hadronic scale of the order of a few hundred MeV. More precisely, p^2, $P \cdot p$ and k^2, $P_h \cdot k$ are of hadronic scale.

In the deep inelastic limit the process factorizes. For the $H \to q$ part one can integrate over p^-. One can then analyze the Dirac content of the projections [9]

$$\Phi^{[\Gamma]}(x_B, \boldsymbol{p}_T) = \frac{1}{2}\int dp^- \; Tr(\Phi\,\Gamma)\Big|_{p^+ = x_B P^+,\; \boldsymbol{p}_T} . \tag{10}$$

Using for Φ the constraints imposed by hermiticity, parity and time reversal invariance one obtains for an unpolarized hadron

$$\Phi^{[\gamma^+]} = f_1(x_B, \boldsymbol{p}_T), \tag{11}$$

$$\Phi^{[1]} = \frac{M}{P^+}\, e(x_B, \boldsymbol{p}_T), \tag{12}$$

$$\Phi^{[\gamma^i]} = \frac{p_T^i}{P^+}\, f^\perp(x_B, \boldsymbol{p}_T), \tag{13}$$

where f_1 is the well-known twist-two quark distribution function, in which we have kept the $\boldsymbol{p}_T$-dependence and e and $f^\perp$ are the twist-three profile functions discussed in ref. [4] and [5], respectively. The twist of the profile functions is obtained from the power $1/(P^+)^{t-2}$ appearing in the correlation function. Note that only after integrating over transverse momenta and expansion in terms of local operators a comparison with the twist known from the operator product expansion can be made.

Similarly, for the $q \to h$ part one can integrate over k^+ and analyze the Dirac content of the projections

$$\Delta^{[\Gamma]}(z, \boldsymbol{k}_T) = \frac{1}{4z}\int dk^+ \; Tr(\Delta\,\Gamma)\Big|_{k^- = P_h^-/z,\; \boldsymbol{k}_T} . \tag{14}$$

As the definition of Δ involves states $|P_h, X\rangle$, which are out-states, one cannot use constraints from time reversal invariance. As a consequence one obtains as the most general Dirac content

$$\Delta^{[\gamma^-]} = D_1(z, -z\boldsymbol{k}_T), \tag{15}$$

$$\Delta^{[i\sigma^{i-}\gamma_5]} = \frac{\epsilon_T^{ij} k_{Tj}}{M_h}\, H_1^\perp(z, -z\boldsymbol{k}_T), \tag{16}$$

$$\Delta^{[1]} = \frac{M_h}{P_h^-}\,E(z,-z\boldsymbol{k}_T), \tag{17}$$

$$\Delta^{[\gamma^i]} = \frac{k_T^i}{P_h^-}\,D^\perp(z,-z\boldsymbol{k}_T), \tag{18}$$

$$\Delta^{[i\sigma^{ij}\gamma_5]} = \frac{M_h\,\epsilon^{ij}}{P_h^-}\,H(z,-z\boldsymbol{k}_T), \tag{19}$$

where D_1 is the well-known twist-two quark fragmentation function with the $\boldsymbol{k}_T$-dependence kept. This dependence translates into a dependence on the transverse momenta of produced hadrons. Furthermore a time reversal odd twist-two fragmentation function $H_1^\perp$ appears, which can only be measured when the dependence on the transverse momenta is measured, e.g. in azimuthal asymmetries. This function $H_1^\perp$ has been discussed in ref. [10] for polarized leptoproduction as a means to probe transversely polarized quarks. At the twist-three level one finds the profile functions E (ref. [4]), $D^\perp$ (ref. [5]) and another time reversal odd function H.

One can gain some insight in the nature of these time reversal odd functions by considering the case that hX would have only one possible channel. Using $|P_h,X;out\rangle = e^{2i\delta_{hX}}\,|P_h,X;in\rangle$ (see [11]), the constraints from time-reversal then would imply that the functions D_1, E and $D^\perp$ are proportional to $\cos\delta_{hX}$, while the functions $H_1^\perp$ and H are proportional to $\sin\delta_{hX}$.

The twist-two functions are the ones that appear if one evaluates the result for the hadronic tensor in leading order. This yields

$$2M\,\mathcal{W}_{\mu\nu} = \left(\frac{\tilde{P}_\mu\tilde{P}_\nu}{\tilde{P}^2}\right)\,2z\,I[f_1\,D_1], \tag{20}$$

where $\tilde{P} = P - (P\cdot q/q^2)\,q$, and $\tilde{P}^2 = Q^2/4x^2$, while the convolution $I[f_1\,D_1]$ is given by

$$I[f_1\,D_1] = \int d^2p_\perp\,d^2k_\perp\,\delta^2(\boldsymbol{p}_\perp + \boldsymbol{q}_T - \boldsymbol{k}_\perp)\,f_1(x_{\scriptscriptstyle B},\boldsymbol{p}_\perp)\,D_1(z,-z\boldsymbol{k}_\perp). \tag{21}$$

As illustration we will consider the situation in which

$$f(x_{\scriptscriptstyle B},\boldsymbol{p}_\perp) = f(x_{\scriptscriptstyle B},0)\,\exp(-R_H^2\boldsymbol{p}_\perp^2), \tag{22}$$

$$D(z,-z\boldsymbol{k}_\perp^2) = D(z,0)\,\exp(-R_h^2\boldsymbol{k}_\perp^2). \tag{23}$$

In that case the above integral can be written as

$$I[f_1\,D_1] = \frac{\pi}{R_H^2+R_h^2}\,\exp\left(-\frac{\boldsymbol{q}_T^2R_H^2R_h^2}{R_H^2+R_h^2}\right)\,f_1(x_{\scriptscriptstyle B},\boldsymbol{0}_T)\,D_1(z,\boldsymbol{0}_T). \tag{24}$$

Integrating over the perpendicular momenta of the produced hadron, i.e. over $\boldsymbol{q}_T$ one obtains the familiar factorized form $f_1(x_{\scriptscriptstyle B})\,D_1(z)$, where $f_1(x_{\scriptscriptstyle B}) = \int d^2p_T\,f_1(x_{\scriptscriptstyle B},\boldsymbol{p}_T)$ is the lightcone momentum distribution and $D_1(z) = \int d^2P_{h\perp}D_1(z,\ \boldsymbol{P}_{h\perp})$ is the fragmentation function. Finally it should be noted that everywhere the summation over quark flavors has been suppressed. Together with the antiquark part, it can easily be reinstated by adding a summation over flavors of quarks *and* antiquarks weighted with the charge squared (e_i^2) and labeling the functions as $f_1^{i/H}$ and $D_1^{h/i}$ respectively.

The Born contribution to $\mathcal{W}_{\mu\nu}$ also gives a contribution at subleading order (proportional to $1/Q$). These involve the twist-three functions defined above. At the same order, however, also contributions from diagrams with gluons, as shown in Fig. 2 for quarks, must be included. They give in addition to the result in Eq. 6 contributions that involve quark - quark - gluon correlation

functions. It turns out that they in the gauges $A^+ = 0$ (for $H \to q$ part) and $A^- = 0$ (for $q \to h$ part) at the twist-three level contribute through the bilocal matrix elements

$$F^\alpha_{ij}(p) = \frac{1}{(2\pi)^4} \int d^4x\ e^{i\,p\cdot x} \langle P|\overline{\psi}_j(0) A^\alpha_T(x)\psi_i(x)|P\rangle. \tag{25}$$

$$M^\alpha_{ij}(k) = \frac{1}{(2\pi)^4} \sum_X \int d^4x\ e^{i\,k\cdot x} \langle 0|A^\alpha_T(x)\psi_i(x)|P_h, X\rangle\langle P_h, X|\overline{\psi}_j(0)|0\rangle. \tag{26}$$

Using the equations of motion for the quark fields, these can be reexpressed in the quark - quark correlation functions Φ and Δ.

At this point it is appropriate to mention that in order to have a color gauge invariant definition one must include a color link in the bilocal quark-quark correlation functions in Eqs 7 and 8 and similarly in the quark-quark-gluon correlation functions in Eq. 25 and 26. For correlation function integrated over k^- *and* $\boldsymbol{k}_T$ one is only sensitive to the nonlocality in the x^- direction. In that case a link $L(0,x) = \mathcal{P}\ \exp[ig\int_0^x ds\cdot A(s)]$ with a straight path can be inserted in the definition of profile functions. Such a link operator becomes unity in the gauge $A^+ = 0$. When transverse momentum is observed and one only integrates over k^-, one also becomes sensitive to separation in the transverse direction, although still $x^+ = 0$. In that case one also needs to fix the gauge freedom affecting A_T. This requires a choice of boundary conditions [12] for A_T and correspondingly a particular choice of path in the link operator. Although it is not hard to find some way to do this in such a way that the link operator reduces to unity after fixing the gauge, it remains to be proven that this can be combined with a proof for factorization at the twist three level.

The full (electromagnetically gauge invariant) result of the diagrammatic expansion up to $\mathcal{O}(1/Q)$ reads

$$\begin{aligned}
2M\,\mathcal{W}_{\mu\nu} \;=\; & \int d^2p_T\, d^2k_T\, \delta^2(\boldsymbol{p}_T + \boldsymbol{q}_T - \boldsymbol{k}_T) \\
& \times\Bigg[\left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} + \frac{\tilde{P}_\mu\tilde{P}_\nu}{\tilde{P}^2}\right) 2z\, f_1\, D_1 \\
& + \frac{(\tilde{P}_\mu p_{\perp\nu} + \tilde{P}_\nu p_{\perp\mu})}{Q^2}\, 8x_B z\, f^\perp\, D_1 \\
& + \frac{(\tilde{P}_\mu k_{\perp\nu} + \tilde{P}_\nu k_{\perp\mu})}{Q^2}\, 8z\, f_1 \left(\frac{1}{z}\, D^\perp - D_1\right) \\
& + i\,\frac{(\tilde{P}_\mu k_{\perp\nu} - \tilde{P}_\nu k_{\perp\mu})}{Q^2}\, 8z\,\frac{M}{M_h}\left(x_B e - \frac{m}{M}\, f_1\right) H_1^\perp\Bigg],
\end{aligned} \tag{27}$$

where $p_\perp$ is constructed from the vector that in the frame where the hadrons (H and h) are collinear is $p_T = [0, 0, \boldsymbol{p}_T]$ by subtracting the projection along q, i.e. $p_\perp = p_T + (\boldsymbol{p}_T\cdot\boldsymbol{q}_T/q^2)\, q$. This vector is orthogonal to q. Note, however that neglecting $1/Q^2$ contributions, the perpendicular components are the same, i.e. $\boldsymbol{p}_T = \boldsymbol{p}_\perp$.

In order to investigate the observable consequences of the various terms in the hadronic tensor, one needs to consider the contraction with the leptonic tensor. This gives

$$L^{\mu\nu}\left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} + \frac{\tilde{P}_\mu\tilde{P}_\nu}{\tilde{P}^2}\right) = \frac{4Q^2}{y^2}\left(\frac{y^2}{2} + 1 - y\right), \tag{28}$$

$$L^{\mu\nu}\left(\frac{\tilde{P}_\mu p_{\perp\nu} + \tilde{P}_\nu p_{\perp\mu}}{Q^2}\right) = -\frac{4Q^2}{y^2}\,\frac{|\boldsymbol{p}_T|}{Q}\,\frac{(2-y)\sqrt{1-y}}{2x}\,\cos\phi_p, \tag{29}$$

$$L^{\mu\nu}\left(i\frac{\tilde{P}_\mu k_{\perp\nu} - \tilde{P}_\nu k_{\perp\mu}}{Q^2}\right) = -\frac{4Q^2}{y^2}\,\frac{|\boldsymbol{k}_T|}{Q}\,\frac{y\sqrt{1-y}}{2x}\,\lambda\sin\phi_k, \tag{30}$$

where the azimuthal angles are those between the transverse vectors and the (lepton) scattering plane, e.g. $\boldsymbol{p}_T \cdot \boldsymbol{q}_T = |\boldsymbol{p}_T|\,|\boldsymbol{q}_T| \cos\phi_p$.

As the most general result one can consider the case where one measures the perpendicular direction of the struck quark from the jet direction ($\boldsymbol{p}_\perp$) and the momentum of one specific hadron (h) belonging to this jet, characterized by the lightcone momentum fraction z and the perpendicular momentum $\boldsymbol{p}_{h\perp}$. One obtains (using $\boldsymbol{k}_\perp = \boldsymbol{p}_\perp -$ boldmath $P_{h\perp}/z$)

$$\begin{aligned}
\frac{d\sigma^{(\ell+H\rightarrow\ell'+h+\mathrm{jet}+X)}}{dx_B dy dz d^2\boldsymbol{P}_{h\perp} d^2\boldsymbol{p}_\perp} = \frac{4\pi\alpha^2\, s}{Q^4} \Bigg\{ &\left(\frac{y^2}{2}+1-y\right) x_B f_1\, D_1 \\
&+ 2(2-y)\sqrt{1-y}\,\cos\phi_h\, \frac{|\boldsymbol{P}_{h\perp}|}{zQ} \left[x_B f_1 \left(\frac{1}{z} D^\perp - D_1\right)\right] \\
&+ 2(2-y)\sqrt{1-y}\,\cos\phi_j\, \frac{|\boldsymbol{p}_\perp|}{Q} \left[-\frac{x_B}{z}\, f_1\, D^\perp + x_B \left(f_1 - x_B f^\perp\right) D_1 \right] \\
&+ 2y\sqrt{1-y}\,\lambda\,\sin\phi_h\, \frac{|\boldsymbol{P}_{h\perp}|}{zQ} \left[\frac{M x_B}{M_h} \left(x_B e - \frac{m}{M}\, f_1 \right) H_1^\perp \right] \\
&- 2y\sqrt{1-y}\,\lambda\,\sin\phi_j\, \frac{|\boldsymbol{p}_\perp|}{Q} \left[\frac{M x_B}{M_h} \left(x_B e - \frac{m}{M}\, f_1 \right) H_1^\perp \right] \Bigg\},
\end{aligned} \tag{31}$$

where the $H \to q$ profile functions (f_1, $f^\perp$ and e) depend on x_B and $\boldsymbol{p}_\perp^2$, while the $q \to h$ profile functions (D_1, $D^\perp$ and $H_1^\perp$) depend on z and $(\boldsymbol{P}_{h\perp} - z\boldsymbol{p}_\perp)^2$.

The theoretically simplest results are the ones obtained by integrating over the momenta of produced hadrons [5]. Integrating over the perpendicular momenta one has

$$\begin{aligned}
\frac{d\sigma^{(\ell+H\rightarrow\ell'+h+\mathrm{jet}+X)}}{dx_B dy dz d^2\boldsymbol{p}_\perp} &= \frac{4\pi\alpha^2\, s}{Q^4} \Bigg\{ \left(\frac{y^2}{2}+1-y\right) x_B f_1(x_B, \boldsymbol{p}_\perp)\, D_1(z) \\
&\qquad - 2(2-y)\sqrt{1-y}\,\cos\phi_j\, \frac{|\boldsymbol{p}_\perp|}{Q}\, x_B^2 f^\perp(x_B, \boldsymbol{p}_\perp)\, D_1(z) \Bigg\}.
\end{aligned} \tag{32}$$

This shows that the ratio $x_B f^\perp(x_B, \boldsymbol{p}_\perp)/f_1(x_B, \boldsymbol{p}_\perp)$ can be obtained most cleanly from the azimuthal $\langle \cos\phi_j\rangle$ asymmetry of the produced jet. Note that this ratio is an extension of the naive parton result of unity [13]. A calculation in the bag model is shown in ref. [14].

The measurement of the time reversal odd fragmentation function $H_1^\perp$ is possible from the general cross section given above. However, the detection of a jet in the forward direction will be hard in view of the fact that one is interested in a twist-three piece ($\propto 1/Q$). This makes it more realistic to consider $\ell + H \to \ell' + h + X$. Only convolutions of profile functions from the distribution and fragmentation part will appear in this case. We will consider again the approximations for the transverse momentum dependence as in Eqs. 22 and 23 and express the results in convolutions I as defined in Eq. 21. The result for the cross section for electroproduction of a hadron h up to $\mathcal{O}(1/Q)$ becomes

$$\begin{aligned}
\frac{d\sigma^{(\ell+H\rightarrow\ell'+h+X)}}{dx_B dy dz d^2\boldsymbol{p}_\perp} = \frac{4\pi\alpha^2\, s}{Q^4} \Bigg\{ &\left(\frac{y^2}{2}+1-y\right) x_B I[f_1\, D_1] \\
&+ 2(2-y)\sqrt{1-y}\,\cos\phi_h\, \frac{|\boldsymbol{P}_{h\perp}|}{zQ} \left(\frac{R_h^2\, x_B}{R_H^2 + R_h^2}\, I\left[f_1 \left(D_1 - \frac{1}{z} D^\perp \right)\right] - \frac{R_H^2\, x_B^2}{R_H^2 + R_h^2}\, I[f^\perp D_1] \right) \\
&+ 2y\sqrt{1-y}\,\lambda \sin\phi_h\, \frac{|\boldsymbol{P}_{h\perp}|}{zQ} \left(\frac{M x_B}{M_h} \frac{R_h^2}{R_H^2 + R_h^2}\, I\left[\left(\frac{m}{M}\, f_1 - x_B e \right) H_1^\perp \right] \right) \Bigg\}.
\end{aligned} \tag{33}$$

The time reversal odd fragmentation function $H_1^\perp$ is a leading twist fragmentation function which appears as a consequence of the fact that the states $|p_h, X\rangle$ in the definition of Δ are out-states. This means that it is nonzero because of the interactions of the outgoing hadron h. Furthermore, this time reversal odd structure function appears in the cross section as part of a product that also contains the profile function e. This chirally odd function has been discussed in ref. [4] and an estimate using the bag model has been given. We note that the combination that appears in the cross sections corresponds to a pure quark-quark-gluon correlation function, to be precise $i(Mx_B\, e - m\, f_1) = (1/2)\int dk^-\, \mathrm{Tr}\,(g\, F_\alpha\, \sigma^{\alpha+})$.

In principle the extraction of the $\sin\phi_h$ asymmetry for pions does not seem to be extremely difficult. One does not need target polarization, while the result is proportional to the lepton helicity. The azimuthal structure in the quark transverse momenta plays the key role in this. The precise $|\,\boldsymbol{p}_\perp\,|$ dependence and dilution of it by QCD corrections (Sudakov effects [15]) are less relevant. It is important to emphasize that the $\boldsymbol{p}_\perp$-dependence that we have discussed is the low-momentum range, say below 1 GeV. Here the $\boldsymbol{p}_\perp$-dependence is dominated by the intrinsic transverse momentum of the profile functions, rather than by perturbative QCD processes [16] that dominate the high transverse momenta [17, 18, 19]. The fact that one deals with a twist three structure function furthermore requires not too large values for the momentum transfer Q. It is important, however, to have good particle identification and sufficient azimuthal resolution in the forward direction.

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